Name: ______________________

MA 1118 - Multivariable Calculus
Quiz 4 - Quarter I - AY 02-03

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. No notes or tables permitted.

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1. (15 points) Given

$$f(x, y) = x^2 y + \cos(xy^2)$$

find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$

solution:

To find $\frac{\partial f}{\partial x}$, we treat all other variables (in this case y) as constants, and then just take an ordinary derivative, i.e.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[x^2 y + \cos(xy^2)\right] = 2xy - y^2 \sin(xy^2)$$

where we differentiated the first term using just the x^n rule, and the second term using the chain rule.

Proceeding similarly then

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left[2xy - y^2 \sin(xy^2)\right] \\ &= 2y - y^2\left(y^2 \cos(xy^2)\right) = 2y - y^4 \cos(xy^2)\end{aligned}$$

Finally

$$\begin{aligned}\frac{\partial^2 f}{\partial y \partial x} &= \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left[2xy - y^2 \sin(xy^2)\right] \\ &= 2x - 2y \sin(xy^2) - y^2\left(2xy \cos(xy^2)\right) \\ &= 2x - 2y \sin(xy^2) - 2xy^3 \cos(xy^2)\end{aligned}$$

2. (5 points) Identify and sketch the domain of: $f(x, y) = \ln(x^2 - y^2)$

solution:

Since this is a composition with the logarithm, the domain should fairly clearly seen to be:

$$x^2 - y^2 > 0 \quad \Longrightarrow \quad x^2 > y^2 \quad \Longrightarrow \quad |x| > |y|$$

This is precisely the region shaded below: